APPENDIX

UNIVERSITY PHYSICS

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13-7 Stokes' law

When an ideal fluid of zero viscosity flows past a sphere, or when a sphere moves through a stationary fluid, the streamlines form a perfectly symmetrical pattern around the sphere, as shown in Fig. 13-2a. The pressure at any point on the upstream hemispherical surface is exactly the same as that at the corresponding point on the downstream face, and the resultant force on the sphere is zero. If the fluid has viscosity, however, there will be a viscous drag on the sphere. (A viscous drag is experienced by a body of *any* shape, but only for a sphere is the drag readily calculable.)

We shall not attempt to derive the expression for the viscous force directly from the laws of flow of a viscous fluid. The only quantities on which the force can depend are the viscosity η of the fluid, the radius r of the sphere, and its velocity v relative to the fluid. A complete analysis shows that the force F is given by

$$F = 6\pi\eta r v. \tag{13-19}$$

This equation was first deduced by Sir George Stokes in 1845 and is called *Stokes' law*. We have already used it in Sec. 4-6 (Example 9) to study the motion of a sphere falling in a viscous fluid. At that point it was necessary to know only that the viscous force on a given sphere in a given fluid is proportional to the relative velocity.

A sphere falling in a viscous fluid reaches a *terminal velocity* v_T at which the viscous retarding force plus the buoyant force equals the weight of the sphere. Let ρ be the density of the sphere and ρ' the density of the fluid. The weight of the sphere is then $(4/3)\pi r^3\rho g$, and the buoyant force is $(4/3)\pi r^3\rho' g$; when the terminal velocity is reached, the total force is zero and

$$\tfrac{4}{3}\pi r^3\rho' g + 6\pi\eta r v_T = \tfrac{4}{3}\pi r^3\rho g,$$

or

$$v_T = \frac{2}{9}\frac{r^2 g}{\eta}(\rho - \rho'). \tag{13-20}$$

When the terminal velocity of a sphere of known radius and density is measured, the viscosity of the fluid in which it is falling can be found from the equation above. Conversely, if the viscosity is known, the radius of the sphere can be determined by measuring the terminal velocity. This method was used by Millikan to determine the radius of very small electrically charged oil drops (used to measure the electrical charge of an inidividual electron) by observing free fall in air.

Even for nonspherical bodies, a relation of the form of Eq. (13-19) holds, with a different numerical coefficient. Biologists call the terminal velocity the *sedimentation velocity*, and experiments with sedimentation can give useful information concerning very small particles. It is often useful to increase the terminal velocity by spinning the sample in a centrifuge, which greatly increases the effective acceleration of gravity.

13-8 Reynolds number

When the velocity of a fluid flowing in a tube exceeds a certain critical value (which depends on the properties of the fluid and the diameter of

the tube), the nature of the flow becomes extremely complicated. Within a very thin layer adjacent to the tube walls, called the *boundary layer,* the flow is still laminar. The flow velocity in the boundary layer is zero at the tube walls and increases uniformly throughout the layer. The properties of the boundary layer are of the greatest importance in determining the resistance to flow, and the transfer of heat to or from the moving fluid.

Beyond the boundary layer, the motion is highly irregular. Random local circular currents called *vortices* develop within the fluid, with a large increase in the resistance to flow. Flow of this sort is called *turbulent.*

Experiment indicates that a combination of four factors determines whether the flow of a fluid through a tube or pipe is laminar or turbulent. This combination is known as the *Reynolds number,* N_R, and is defined as

$$N_R = \frac{\rho v D}{\eta}. \tag{13-21}$$

where ρ is the density of the fluid, v the average forward velocity, η the viscosity, and D the diameter of the tube. (The average velocity is defined as the uniform velocity over the entire cross section of the tube, which would result in the same volume rate of flow.) The Reynolds number, $\rho v D/\eta$, is a *dimensionless* quantity and has the same numerical value in any consistent system of units. For example, for water at 20°C flowing in a tube of diameter 1 cm with an average velocity of 10 $\mathrm{cm \cdot s^{-1}}$, the Reynolds number is

$$N_R = \frac{\rho v D}{\eta} = \frac{(1\ \mathrm{g \cdot cm^{-3}})(10\ \mathrm{cm \cdot s^{-1}})(1\ \mathrm{cm})}{0.01\ \mathrm{dyn \cdot s \cdot cm^{-2}}} = 1000.$$

Had the four quantities been expressed originally in SI units, the same value of 1000 would have been obtained.

A variety of experiments have shown that when the Reynolds number is less than about 2000 the flow is laminar, whereas above about 3000 the flow is turbulent. In the transition region between 2000 and 3000 the flow is unstable and may change from one type to the other. Thus for water at 20°C flowing in a tube 1 cm in diameter, the flow is laminar when

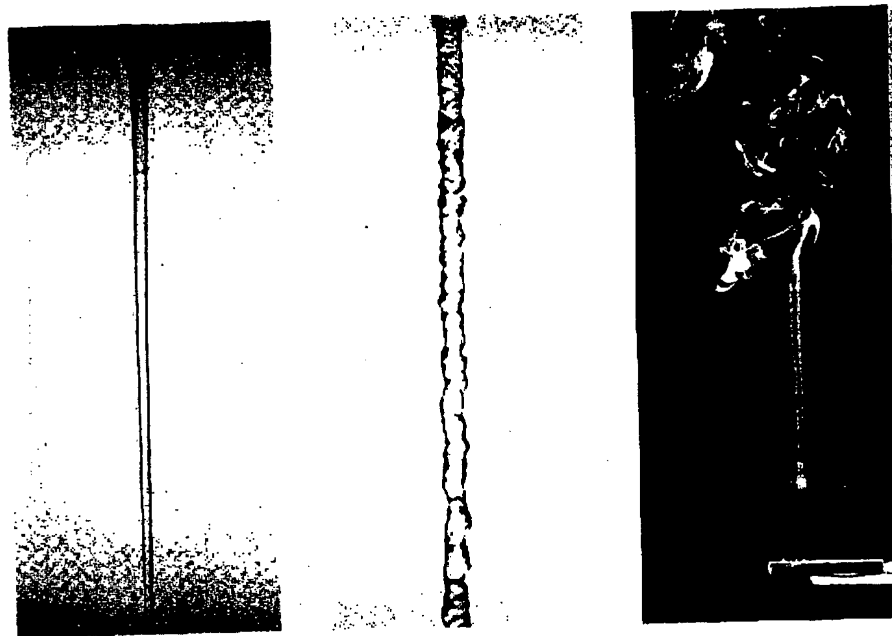
$$\frac{\rho v D}{\eta} \leq 2000,$$

or when

$$v \leq \frac{(2000)(0.01\ \mathrm{dyn \cdot s \cdot cm^{-2}})}{(1\ \mathrm{g \cdot cm^{-3}})(1\ \mathrm{cm})} = 20\ \mathrm{cm \cdot s^{-1}}.$$

Above about 30 $\mathrm{cm \cdot s^{-1}}$ the flow is turbulent. If air at the same temperature were flowing at 30 $\mathrm{cm \cdot s^{-1}}$ in the same tube, the Reynolds number would be

$$N_R = \frac{(0.0013\ \mathrm{g \cdot cm^{-3}})(30\ \mathrm{cm \cdot s^{-1}})(1\ \mathrm{cm})}{181 \times 10^{-6}\ \mathrm{dyn \cdot s \cdot cm^{-2}}} = 215.$$



13-15 (a) Laminar flow. (b) Turbulent flow. (c) First laminar, then turbulent.

(a) (b) (c)

Since this is much less than 3000, the flow would be laminar and would not become turbulent unless the velocity were as great as 420 $cm \cdot s^{-1}$.

The distinction between laminar and turbulent flow is shown in the photographs of Fig. 13-15. In (a) and (b) the fluid is water and in (c) air and smoke particles.

The Reynolds number of a system forms the basis for the study of the behavior of real systems through the use of small scale models. A common example is the wind tunnel, in which one measures the aerodynamic forces on a scale model of an aircraft wing. The forces on a full-size wing are then deduced from these measurements.

Two systems are said to be *dynamically similar* if the Reynolds number, $\rho v D/\eta$, is the same for both. The letter D may refer, in general, to any dimension of a system, such as the span or chord of an aircraft wing. Thus the flow of a fluid of given density ρ and viscosity η, about a half-scale model, is dynamically similar to that around the full-size object if the velocity v is twice as great.

Questions

13-1 Is the continuity relation, Eq. (13-1), valid for compressible fluids? If not, is there a similar relation that *is* valid?

13-2 If the velocity at each point in space in steady-state fluid flow is constant, how can a fluid particle accelerate?

13-3 Whenever possible, airplanes take off and land heading into the wind. Why?

13-4 Does the "lift" of an airplane wing depend on altitude?

13-5 How does a baseball pitcher give the ball the spin that makes it curve? Can he make it curve in either direction? Does it matter whether he is righthanded or lefthanded? What is a spitball? Why is it illegal?

13-6 When a car on a highway is passed by a large truck,